

**Definition:**

A differential equation is one which contains differential coefficients or differentials

Types:

1. Ordinary: ordinary differential equations are those which involve the differential coefficient with respect to a single independent variable.

Eg: $dy = \tan x \, dx$

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0$$

2. Partial: Partial differential equations are those in which there are at least two or more independent variables and partial differential coefficients with respect to any one of them

Eg: $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = z^2$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

ORDER & DEGREE OF DIFFERENTIAL EQUATION

The order of a differential equation is the order of the highest derivative appearing in it.

The equation

$dy = \tan x \, dx$ is of first order and

$$\frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + 2y = 0 \text{ is of second order}$$

The degree of a differential equation is the degree of the derivative of the highest order, which is found in the equation, after it is cleared of radicals and fractions.

The equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$

Is of second degree and second order.

Note: 1. The solution of a differential equation is also called its primitive

2. The solution of a differential equation which involves as many arbitrary constants as its order is called the most general solution. Thus the most general solution of a n^{th} order differential equation will contain n arbitrary constants.

3. To form a differential equation corresponding to a given solution containing arbitrary constants, we differentiate it as many times as are necessary and then eliminate the constants.

Eg: The differential equation of simple harmonic motion is given by

$$X = A \cos(nt + h), \text{ A \& h are constants}$$

$$\Rightarrow dx / dt = -An \times \sin(nt + h)$$

$$\Rightarrow d^2x / dt^2 = -n^2 An \cos(nt + h) = -n^2x$$

$$\Rightarrow d^2x / dt^2 + n^2x = 0 \text{ is the required equation}$$

2. The differential equation of all circles of radius 'a' is
the equation of circle with (h, k) as centre & radius 'a' is
 $(x - h)^2 + (y - k)^2 = a^2$

On differentiating twice i.e.,

$$2(x - h) + 2(y - k) dy/dx = 0 ; (x - h) + (y - k) dy/dx = 0$$

$$\Rightarrow 1 + (y - k) d^2y / dx^2 + (dy/dx)^2 = 0$$

(Contd...2)

→ ACE →

:: 2 ::

→ ACE →

$$(y-k) = - \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$

$$\text{and } (x-h) = -(y-k) \frac{dy}{dx} = \frac{dy}{dx} \frac{1 + \left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$

Substituting (y-k) & (x-h) in equation (1) we get

$$\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2 \quad \text{is the required equation.}$$

3. The differential equation of all circles which pass through the origin and whose centres are on the x-axis is

Sol: Let the centre of one such circle be at the point (a, 0)

$$(x-a)^2 + (y-0)^2 = a^2$$

$$\Rightarrow x^2 + y^2 - 2ax = 0$$

$$\Rightarrow 2x + 2y \frac{dy}{dx} - 2a = 0 \Rightarrow 2a = 2x + 2y \frac{dy}{dx}$$

∴ The differential equation formed is

$$X^2 + Y^2 - x \left[2x + 2y \frac{dy}{dx} \right] = 0$$

$$Y^2 - X^2 - 2xy \frac{dy}{dx} = 0$$

FIRST ORDER NON-LINEAR ORDINARY DIFFERENTIAL EQUATIONS:

1. Equations where variables are separable:

If in an equation it is possible to collect all functions of 'x' and dx on one side and all the functions of 'y' & dy on the other side, then the variables are said to be separable

General form $f(y) dy = g(x) dx$

On integrating both sides, we get

$\int f(y) dy = \int g(x) dx + C$ as its solution

Eg : Solve $xy \frac{dy}{dx} = 1 + x + y + xy$

Sol. $xy \frac{dy}{dx} = (1+x) + y(1+x) = (1+x)(1+y)$

$$\frac{y}{1+y} dy = \left(\frac{x+1}{x} \right) dx$$

$$\left(1 - \frac{1}{1+y} \right) dy = (1 + 1/x) dx$$

On integrating both sides we get

$$\int \left(1 - \frac{1}{1+y} \right) dy = \int (1 + 1/x) dx$$

$$y - \log(1+y) = x + \log(x) + C$$

$$(y - x) - \log x(1+y) = C$$

2. $dy/dx = xe^{y-x^2}$ and $y(0) = 0$

$$dy/dx = x e^y / e^{x^2}$$

$$\Rightarrow e^{-y} dy = x e^{-x^2} dx \Rightarrow \int e^{-y} dy = \int e^{-x^2} \left(\frac{-2x}{-2} \right) dx$$

$$e^{-y} / -1 = e^{-x^2} / -2 + C$$

(Contd...3)

→ ACE →

:: 3 ::

→ ACE →

As $y(0) = 0$,

$$-e^{-0} = -e^{-0} / 2 + C$$

$$-1 = -1/2 + c \Rightarrow c = -1 + 1/2 = -1/2$$

Required equation,

$$-e^{-y} = \frac{e^{-x^2}}{-2} - 1/2$$

$$\Rightarrow e^{-y} = \frac{e^{-x^2} + 1}{2} \Rightarrow 2e^{-y} - e^{-x^2} - 1 = 0$$

Note : The above problem in which initial condition $y(0) = 0$ is given, it is called as initial value problem.

2. Homogeneous Equations

The equations of the form $dy / dx = f(x, y) / g(x, y)$ is homogeneous when $f(x, y)$ & $g(x, y)$ are homogeneous functions of the same degree in x & y .

To solve this type of equation

i) put $y = vx \Rightarrow dy/dx = v + x dv/dx$

ii) separate the variables v & x , and integrate

Eg: 1- Solve $(x^2 - y^2) dx = 2xy dy$

Sol: $\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$ is homogeneous in x & y

Put $y = vx \Rightarrow dy/dx = v + x dv/dx$

$$v + x \frac{dv}{dx} = \frac{x^2 - v^2x^2}{2x vx} = \frac{1 - v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{1 - v^2}{2v} - v = \frac{1 - 3v^2}{2v}$$

$$\frac{2v}{1 - 3v^2} dv = dx / x$$

Integrating on both sides,

$$\int \frac{2v dv}{1 - 3v^2} = \int dx/x$$

$$\Rightarrow -1/3 \log(1 - 3v^2) = \log x + c$$

$$\Rightarrow 3 \log x + \log(1 - 3v^2) = -3c$$

$$\log x^3(1 - 3v^2) = -3c \log e$$

$$\Rightarrow x^3(1 - 3v^2) e^{-3c} = k$$

$$\Rightarrow x(x^2 - 3y^2) = k$$

3. Linear Equations :

A differential equation is said to be linear if the dependent variable and its differential coefficients occur in the first degree and not multiplied together.

Standard form : $dy/dx + Py = Q$

Where P & Q are functions of x

Solution : $y(I.F) = \int Q(I.F)dx + c$

Where I.F is called integrating factor & $I.F = e^{\int P dx}$

Eg : 1. Solve $x \log x dy/dx + y = \log x^2$

Sol: $dy/dx + y / x \log x = 2/x$

$$P = 1 / x \log x, Q = 2/x$$

$$I.F = e^{\int 1/x \log x dx} = e^{\log(\log x)} = \log x$$

Solution if $y(I.F) = \int Q(I.F) dx + c$

$$y \log x = \int 2/x \log x dx + c$$

$$y \log x = (\log x)^2 + c$$

$$\Rightarrow y = \log x + c / \log x$$

(Contd...4)

→ ACE →

:: 4 ::

→ ACE →

4. Exact differential equations:

a) A differential equation of the form

$$M(x, y)dx + N(x, y) dy = 0$$

is said to be exact if its left hand member is the exact differential of some function $u(x, y)$ i.e.,

$$du = M dx + N dy = 0$$

∴ Its solution is $u(x, y) = C$

b) The necessary and sufficient condition for the differential equation $M dx + N dy = 0$ to be exact is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

c) The solution of $M dx + N dy = 0$ is

$$\int M dx + \int (\text{terms of } N \text{ not containing } x) dy = C, (y \text{ is constant})$$

$$\text{provided } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Eg : Solve $(y \cos x + \sin y + y) dx + (\sin x + x \cos y + x) dy = 0$

Sol: $M dx + N dy = 0$

$$M = y \cos x + \sin y + y$$

$$N = \sin x + x \cos y + x$$

$$\frac{\partial M}{\partial y} = \cos x + \cos y + 1$$

$$\frac{\partial N}{\partial x} = \cos x + \cos y + 1$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Solution is $\int (y \cos x + \sin y + Y) dx + \int 0 dx = c$

$$\Rightarrow y \sin x + x \sin y + xy = C$$

$$\Rightarrow y \sin x + x [\sin y + y] = c$$

Exercise

1. Solve $(xy^2 + x) dx + (yx^2 + y) dy = 0$

2. Solve $\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}$

3. $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$

4. $2xy \frac{dy}{dx} = 3y^2 + x^2$

5. $\frac{dy}{dx} - \frac{2y}{x+1} = (x+1)^3$

6. $e^{-y} \sec^2 y dy = dx + x dy$

7. $(x^2 - ay)dx = (ax - y^2) dy$

8. $Y e^{xy} dx + (x e^{xy} + 2y) dy = 0$

9. $[y(1 + 1/x) + \cos y] dx + (x + \log x - x \sin y) dy = 0$

KEY

1. $(y^2+1)(x^2+1) = c$

3. $y = 2x \tan^{-1}(cx)$

5. $y = (x+1)^2 / 2 + c(x+1)^2$

7. $x^3 + y^3 - 3axy = c$

9. $y(x + \log x) + x \cos y = c$

2. $y \sin y = x^2 \log x + c$

4. $x^2 + y^2 = cx^3$

6. $xe^y - \tan y = c$

8. $e^{xy} + y^2 = c$

(Contd...5)

→ ACE →

:: 5 ::

→ ACE →

Linear Differential Equations :

Definitions:

Linear differential equations are those in which the dependent variable and its derivatives occur only in the first degree and are not multiplied together. Thus the general linear differential equation of the n^{th} order is of the form

$$\frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + p_2 \frac{d^{n-2} y}{dx^{n-2}} = \dots + p_n y = x$$

Where $p_1, p_2, \dots, p_n$ and x are functions of x only

Linear differential equations with constant coefficients are of the form

$$\frac{d^n y}{dx^n} + K_1 \frac{d^{n-1} y}{dx^{n-1}} + K_2 \frac{d^{n-2} y}{dx^{n-2}} = \dots + K_n y = x$$

Where $K_1, K_2, \dots, K_n$ are constants

$$\Rightarrow (D^n + K_1 D^{n-1} + K_2 D^{n-2} + \dots + K_n) y = 0$$

is a differential equation of order ' x '.

The symbolic coefficient

$$D^n + K_1 D^{n-1} + K_2 D^{n-2} + \dots + K_n = 0$$

is called the auxiliary equation (A.E) with $m_1, m_2, m_3, \dots, m_n$ as its roots

Solution of linear differential equation:

The complete solution of the equation is

$$Y = u + v$$

Where 'u' is called complementary function (C.F) and 'v' is called particular integral (P.I)

Complementary function can be found by finding the roots of the auxiliary equation.

C.F may be written as

Roots of A.E

1. Real and different roots m_1, m_2, m_3
2. Two real and equal roots m_1, m_2, m_3
3. A pair of imaginary roots $\alpha \pm i\beta, m_3, \dots$
4. Two pairs of equal imaginary roots $\alpha \pm i\beta, \alpha \pm i\beta, m_5, \dots$

C.F

1. $c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \dots$
2. $(c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x} + \dots$
3. $e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x) + c_3 e^{m_3 x}$
4. $e^{\alpha x} [(c_1 + c_2 x) \cos \beta x + (c_3 + c_4 x) \sin \beta x] + c_5 e^{m_5 x} + \dots$

Inverse Operator:

1. $\frac{1}{f(D)} X$ is that function of 'x' not containing arbitrary constants which when operated upon by $f(D)$ gives X i.e.,

$$f(D) \left(\frac{1}{f(D)} X \right) = X$$

Thus $\frac{1}{f(D)} X$ satisfies the equation $f(D) y = X$ and is therefore called a particular integral

$$2. \frac{1}{D} X = \int X dx$$

$$3. \frac{1}{D - a} X = e^{ax} \int x e^{-ax} dx$$

Rules for Finding Particular Integral

A differential equation,

$$(D^n + K_1 D^{n-1} + K_2 D^{n-2} + \dots + K_n) y = X$$

has its particular integral,

$$\text{P.I. } \frac{1}{(D^n + K_1 D^{n-1} + \dots + K_n)} X = \frac{1}{f(D)} X$$

Case I When $X = e^{ax}$

$$\frac{1}{f(D)} \times e^{ax} = \frac{1}{f(a)} \times e^{ax} \quad (\text{if } f(a) \neq 0)$$

(Contd...6)

→ ACE →

:: 6 ::

→ ACE →

$$\begin{aligned} \text{x. } \frac{1}{f^1(a)} \times e^{ax} & \quad (\text{if } f(a) = 0 \text{ \& } f^1(a) \neq 0) \\ \text{x}^2 \frac{1}{f^{11}(a)} \times e^{ax} & \quad (\text{if } f^{11}(a) \neq 0 \text{ \& } f(a) = f^1(a) = 0) \end{aligned}$$

Case II When $X = \sin(ax+b)$ or $\cos(ax+b)$

$$\begin{aligned} \frac{1}{f(D^2)} \times \sin(ax+b) &= \frac{1}{f(-a^2)} \times \sin(ax+b) \quad (\text{if } f(-a^2) \neq 0) \\ &= x \frac{1}{f(-a^2)} \times \sin(ax+b) \quad (\text{If } f(-a^2) = 0 \text{ \& } f^1(-a^2) \neq 0) \end{aligned}$$

Similarly, $\frac{1}{f(D^2)} \cos(ax+b)$

$$= \frac{1}{f(-a^2)} \times \cos(ax+b) \quad (\text{If } f(-a^2) \neq 0)$$

$$X \frac{1}{f^1(-a^2)} \times \cos(ax+b) \quad (\text{If } f(-a^2) = 0 \text{ \& } f^1(-a^2) \neq 0)$$

$$X^2 \frac{1}{f^{11}(-a^2)} \cos(ax+b) \quad (\text{If } f(-a^2) = f^1(-a^2) = 0 \text{ \& } f^{11}(-a^2) \neq 0)$$

Case III When $X = x^m$

$$\frac{1}{f(D)} \times X^m = [f(D)]^{-1} x^m$$

$[f(D)]^{-1}$ is expanded in ascending powers of D as for the term in D^m and operate on x^m , term by term. Since $(m+1)^{\text{th}}$ and higher derivatives of x^m are zero and we do not consider terms beyond D^m .

Note:

Some useful expansions

$$(1+D)^{-1} = 1 - D + D^2 - D^3 + D^4 - \dots$$

$$(1+D)^{-1} = 1 + D + D^2 + D^3 + D^4 - \dots$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$\sin hx = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$\cos hx = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \dots$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} + \dots$$

$$a^x = 1 + x - \frac{x}{1!} \log a + \frac{x^2}{2!} (\log a)^2 - \frac{x^3}{3!} (\log a)^3 + \dots$$

Case IV $X = e^{nx}V$ & $V = g(x)$

$$\text{P.I} = \frac{1}{f(D)} e^{ax} V = e^{ax} \frac{1}{f(D+a)} V$$

(Contd...7)

→ ACE →

:: 7 ::

→ ACE →

Case V X some other function of X

$$P.I. = \frac{1}{f(D)} X$$

$\frac{1}{f(D)}$ is resolved into partial fractions and operate each partial fraction on & remembering that

$$\frac{1}{D-a} X = e^{ax} \int x e^{-ax} dx$$

Worked Out Examples

1.

$$\text{Solve } \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - 24y = 0$$

Sol: Given $(D^2 + 2D - 24)y = 0$

$$A.E \Rightarrow m^2 + 2m - 24 = 0 \Rightarrow (m+6)(m-4) = 0 \Rightarrow m = -6, 4$$

The roots are real & different

$$\therefore \text{Solution is } y = c_1 e^{-6x} + c_2 e^{4x}$$

2. Solve $(D^2 - 6D + 13)y = 0$

$$\text{sol : } A.E \Rightarrow m^2 - 6m + 13 = 0 \Rightarrow m = 3 \pm 2i$$

The roots are imaginary & occur in conjugate pairs

$$\therefore \text{Solution is } y = e^{3x} (c_1 \cos 2x + c_2 \sin 2x)$$

3. P.I of $(D^2 - 4D + 3)y = e^{2x}$ is

$$P.I = \frac{1}{D^2 - 4D + 3} e^{2x} = \frac{1}{4^2 - 4(2) + 3} e^{2x} = \frac{1}{16 - 8 + 3} e^{2x}$$

$$P.I = -e^{2x}$$

4. P.I of $(D^3 + D^2 + D + 1)y = 2x^3 + 3x^2 - 4x + 5$

$$P.I = \frac{1}{(D^3 - D^2 + D + 1)} (2x^3 + 3x^2 - 4x + 5) = \frac{1}{(1+D)(1+D^2)} (2x^3 + 3x^2 - 4x + 5)$$

$$= (1+D)^{-1} (1+D^2)^{-1} (2x^3 + 3x^2 - 4x + 5)$$

On expanding $(1+D)^{-1}$ & $(1+D^2)^{-1}$ we get

$$= (1-D+D^2-D^3)(1-D^2)(2x^3 + 3x^2 - 4x + 5)$$

$$= (1-D^2-D+D^3+D^2-D^3)(2x^3 + 3x^2 - 4x + 5)$$

$$= (1-D)(2x^2 + 3x^2 - 4x + 5) = (1-D)(2x^2 + 3x^2 - 4x + 5)$$

$$= 2x^2 + 3x^2 - 4x + 5 - 6x^2 - 6x + 4 = 2x^2 + 3x^2 - 10x + 9$$

5. C.F of $(D^3 + 3D^2 + 2D)y = x^2$ is

$$A.E \Rightarrow m^3 + 3m^2 + 2m = 0 \Rightarrow m(m+1)(m+2) = 0 \Rightarrow m = 0, -1, -2$$

6. P.I of $(3D^2 + D - 14)y = 13e^{2x}$, is

$$P.I = \frac{1}{3D^2 + D - 14} 13e^{2x} = \frac{1}{(D-2)(3D+7)}$$

$$\frac{1}{(D-2)} \frac{1}{(3(2)+7)} 13e^{2x} = \frac{1}{(D-2)} e^{2x}$$

$$x = 1/1 e^{2x} = x e^{2x} \quad (f(a) = 0 \text{ \& \; } f'(a) \neq 0) \quad (P.I = x \frac{1}{f'(a)} e^{ax})$$

7. P.I of $(D^2 + D + 1)y = \sin 2x$ is

$$P.I \Rightarrow \frac{1}{(D^2 + D + 1)} \sin 2x = \frac{1}{-4 + D + 1} \sin 2x$$

$$= \frac{1}{D-3} \sin 2x = \frac{D+3}{D^2-9} \sin 2x = \frac{1}{-4-9} (D+2) \sin 2x$$

$$= (1/-13) [2 \cos 2x + 2 \sin 2x]$$

(Contd...8)

→ ACE →

:: 8 ::

→ ACE →

8. P.I of $(D^3 - 3D^2 + 3D - 1)y = x^2 e^x$ is

$$\begin{aligned} \text{P.I} &\Rightarrow \frac{1}{(D-1)^3} e^x x^2 \\ &\left(\because \frac{1}{f(D)} e^{ax} x = e^{ax} \frac{1}{f(D+a)} x \right) \\ &= e^x \frac{1}{(D+1-1)^3} x^2 = e^x \frac{1}{D^3} x^2 \\ &= e^x \frac{1}{D^2} \frac{x^3}{3} = e^x \frac{1}{D} \frac{x^4}{12} \\ &= e^x \frac{x^5}{60} \end{aligned}$$

Exercise

Solve the following

1. $(D^2 + 3D)y = 0$
2. $(D^2 + a^2)y = 0$
3. $(D^4 - m^4)y = 0$
4. $(D^2 - 2D + 1)y = x$
5. $(D^2 + 1)y = x^4$
6. $(D^2 - 3D - 4)y = e^{2x}$

Find P.I of the following

1. $(D^2 + 2D + 2)y = 3 + 2e^x$
2. $(D-1)^2(D-2)y = x^2$
3. $(D^2 + 4D + 4)y = \cosh 2x$
4. $(D^3 - 1)y = (e^x + 1)^2$
5. $(D^3 + 3D^2 + 4D - 2)y = \cos x$
6. $(D^2 + 4)y = 3 \cos^2 x$
7. $(D^2 + 4)y = x \sin x$

KEY

1. $\frac{3}{2} + \frac{2e^x}{5}$
2. $-\left(\frac{x^2}{2} + \frac{5x}{2} + \frac{17}{4}\right)$
3. $\frac{e^{2x}}{32} + \frac{x^2}{4} e^{-2x}$
4. $\frac{e^{2x}}{7} + \frac{2}{3} x e^x - 1$
5. $-1/10[3\sin x + \cos x]$
6. $3/8[1 + x \sin 2x]$
7. $\frac{x \sin x}{3} - 2/9 \cos x$

Linear Equations with Variable Coefficients

An equation of the type given by

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} x \frac{dy}{dx} + a_n y = x \quad \dots(1)$$

Where $a_1, a_2, \dots, a_n$ are constants and x is a function of 'x' and it is called a homogeneous linear differential equation.

The equation (1) can be transformed into linear equation with constant coefficients by introducing a new variable $Z = \log x$

When $Z = \log x$, $dZ/dx = 1/x$

$$\therefore \frac{dy}{dx} = \frac{dy}{dZ} \frac{dZ}{dx} = \frac{1}{x} \frac{dy}{dZ}$$

$$\Rightarrow x \frac{dy}{dx} = \frac{dy}{dZ} = Dy \quad \text{where } D = d/dZ$$

Similarly

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

(Contd...9)

→ ACE →

:: 9 ::

→ ACE →

$$x^3 \frac{d^3 y}{dx^3} = D(D-1)(D-2) \text{ and so on}$$

∴ equation (1) transforms into

$$D(D-1).....(D-n+1) + a_1 D(D-1).....(D-n+2) + + a_{n-1} D + a_n]y = Z$$

Where Z is a function of Z into which X is changed

Eg: 1. Solve $x^3 \frac{d^3 y}{dx^3} + 9x^2 \frac{d^2 y}{dx^2} + 18x \frac{dy}{dx} + 6y = 1/x$

Solutions:

Putting $x = e^z \Rightarrow Z = \log x$

$\theta = d/dz$; $D = d/dx$; $x D = \theta$; $x^2 D^2 = \theta(\theta-1)$; $x^3 D^3 = \theta(\theta-1)(\theta-2)$

$(x^3 D^3 + 9x^2 D^2 + 18x D + 6)y = 1/x$

$[\theta(\theta-1)(\theta-2) + 9\theta(\theta-1) + 18\theta + 6]y = 1/e^z$

$[\theta^3 + 6\theta^2 + 11\theta + 6]y = e^{-z}$

using the earlier methods of solving a differential equation

$y = C.F + P.I$

$= \frac{C_1}{x} + \frac{C_2}{x^2} + \frac{C_3}{x^3} + \frac{1}{2x} \log x$

Exercise

- The order of the differential equation $\left(1 + \frac{dy}{dx}\right)^3 = 3\left(\frac{d^2 y}{dx^2}\right)$ is
a) 1 b) 3 c) 2 d) 4
- The degree of the differential equation $\left(\frac{dy}{dx}\right)^2 = 5\left(\frac{dy}{dx}\right)$ is
a) 1 b) 2 c) 3 d) 0
- The equation $(x^2 + y^2 + x) dx + xy dy = 0$
a) homogenous b) linear in y c) exact d) none of the above
- An integrating factor of $(x^2 + y^2 + 1) dx - 2xy dy = 0$ is
a) $1/x^2$ b) x c) y d) $-x^2$
- An integrating factor of $dx + x dy = e^{-y} \log y dy$ is
a) e^y b) e^x c) x d) e^{-x}
- $2xy dx - (x^2 + y^2 + 1) dx = 0$ is
a) Bernonh equation b) a linear equation
c) an exact equation d) non-homogeneous equation
- The orthogonal properties of $xy = c$ is
a) $x^2 + y^2 = c$ b) $xy = c$ c) $x^2 - y^2 = c$ d) $x^2 = c$
- An integrating factor of $(dy/dx) - (6y/x) = x$ is
a) x^{-6} b) x c) y^{-6} d) x^2
- The equation $(x^2 - 2xy + 3y^2) dx + (4y^3 + 6xy - x^2) dy = 0$ is
a) linear b) homogeneous c) exact d) non exact
- If $ay dx + bx dy = 0$ is exact (a, b constant) then
a) $a - b = 0$ b) $a + b = 0$ c) $x = y$ d) $x + y = 0$
- A differential equation will have
a) only one solution b) infinite number of solutions
c) solutions as many as order d) none of the above

(Contd...10)

→ ACE →

:: 10 ::

→ ACE →

12. For the differential equation $y = x \frac{dy}{dx} + \frac{5}{dy/dx}$; $y^2 = 20x$ is
 a) the general solution b) a particular solution
 c) a singular solution d) none of the above
13. An integrating factor of $y dx - x dy = 0$ is
 a) $1/xy$ b) x^2 c) $1/x$ d) none of the above
14. The differential equation with $(1, x, e^x)$ as the linearly independent solution is
 a) $(D^3 + D + 1)y = 0$ b) $(D^3 - D)y = 0$ c) $(D^2 + D)y = 0$ d) $(D^3 + D^2)y = 0$
15. If the functions $1, e^{3x}$ form the fundamental system of $y'' + ay' + by = 0$ then
 a) $a = 3, b = 0$ b) $a = 0, b = 3$ c) $a = -3, b = 1$ d) $a = 0, b = -3$
16. The differential equation of the family $y = c$ is
 a) $dy/dx = c$ b) $dy/dx = 0$ c) $(dy/dx) + c = 0$ d) none
17. An integrating function of $\frac{dy}{dx} - \frac{5}{x}y = 5x^2$ is
 a) x^5 b) x^{-5} c) x d) x^2
18. The solution of $y' + 5y = 2e^x$, $y(0) = 1$ is
 a) $y = (1/3)(e^x + 2e^{-5x})$ b) $y = (1/3)(e^x - 2e^{-5x})$
 c) $y = (e^x + 2e^{-5x})$ b) none of the above
19. If 1, 2 are roots of auxiliary equation of $f(D)y = X$ then $f(D) = \text{-----}$.
 a) $(D - 1)(D + 2)$ b) $(D - 1)(D - 2)$ c) $D(D - 1)$ d) $D(D - 2)$
20. The particular integral of $\frac{1}{(D^2 + 9)} \cos 3x = \text{-----}$.
 a) $\frac{\sin 3x}{6}$ b) $\frac{x \sin 3x}{6}$ c) $\frac{\cos 3x}{6}$ d) $\frac{x \cos 3x}{6}$

KEY FOR DIFFERENTIAL EQUATIONS

- | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|
| 1. c | 2. c | 3. d | 4. a | 5. a | 6. a | 7. c |
| 8. a | 9. c | 10. a | 11. b | 12. c | 13. a | 14. d |
| 15. a | 16. b | 17. b | 18. a | 19. b | 20. b | |

“ ALL THE BEST ”

204, Rahman Plaza, Opp: Methodist School, Near Taj Mahal Hotel, Fernandez Hospital Lane, Abids, Hyd-1.
 Phones : 4752469 / 6582469 / 6582467